Mathovia — Algebra Cheat Sheet

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Arithmetic and Fraction Rules

Rule	Statement
Distribute a factor	$a(b+c) = ab + ac$
Multiply a fraction by a number	$a \cdot \frac{b}{c} = \frac{ab}{c}$
Fraction divided by a number	$\frac{a/b}{c} = \frac{a}{bc}$
Number divided by a fraction	$\frac{a}{b/c} = \frac{ac}{b}$
Adding unlike fractions	$\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$
Subtracting unlike fractions	$\frac{a}{b} - \frac{c}{d} = \frac{ad-bc}{bd}$
Dividing two fractions	$\frac{a/b}{c/d} = \frac{ad}{bc}$
Canceling a true common factor	$\frac{ab+ac}{a} = b + c,\ a \neq 0$

Exponent Rules

Rule	Statement
Product Rule	$a^m \cdot a^n = a^{m+n}$
Quotient Rule	$\frac{a^m}{a^n} = a^{m-n}$
Power Rule	$(a^m)^n = a^{mn}$
Power of a Product	$(ab)^n = a^n b^n$
Power of a Quotient	$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

Rule	Statement
Zero Exponent	$a^0 = 1,\ a \neq 0$
Negative Exponent	$a^{-n} = \dfrac{1}{a^n}$
Negative Exponent of a Quotient	$\left(\dfrac{a}{b}\right)^{-n} = \left(\dfrac{b}{a}\right)^{n}$
Rational Exponent	$a^{m/n} = \left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$

Radical Rules

Rule	Statement
Root as a fractional exponent	$\sqrt[n]{a} = a^{1/n}$
Product Rule	$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$
Quotient Rule	$\sqrt[n]{\dfrac{a}{b}} = \dfrac{\sqrt[n]{a}}{\sqrt[n]{b}}$
Root of a root	$\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$
Odd-index root of a power	$\sqrt[n]{a^n} = a$ (n odd)
Even-index root of a power	$\sqrt[n]{a^n} = \lvert a \rvert$ (n even)

Inequalities and Absolute Value

Rule	Statement
Add or subtract the same value	If $a < b$, then $a + c < b + c$ and $a - c < b - c$
Multiply or divide by a positive	If $a < b$ and $c > 0$, then $ac < bc$
Multiply or divide by a negative	If $a < b$ and $c < 0$, then $ac > bc$ — direction flips
Absolute value, by definition	$\lvert a \rvert = a$ if $a \geq 0$; $\lvert a \rvert = -a$ if $a < 0$
Product and quotient	$\lvert ab \rvert = \lvert a \rvert \lvert b \rvert$, $\left\lvert \dfrac{a}{b} \right\rvert = \dfrac{\lvert a \rvert}{\lvert b \rvert}$

Rule	Statement
Triangle inequality	$\lvert a+b\rvert \le \lvert a\rvert + \lvert b\rvert$
Solving $\lvert p\rvert = b$ $(b>0)$	$p=-b$ or $p=b$
Solving $\lvert p\rvert < b$ $(b>0)$	$-b<p<b$
Solving $\lvert p\rvert > b$ $(b>0)$	$p<-b$ or $p>b$

Complex Numbers

Rule	Statement
Imaginary unit	$i=\sqrt{-1}$, $i^2=-1$
Square root of a negative	$\sqrt{-a}=i\sqrt{a}$, $a\ge 0$
Add / subtract	Combine real parts and imaginary parts separately
Multiply	FOIL, then replace every i^2 with -1
Conjugate	The conjugate of $a+bi$ is $a-bi$
Conjugate product	$(a+bi)(a-bi)=a^2+b^2$
Modulus	$\lvert a+bi\rvert=\sqrt{a^2+b^2}$
Divide	Multiply the numerator and denominator by the denominator's conjugate

Factoring Formulas

Pattern	Factors as
Difference of squares	$x^2-a^2=(x+a)(x-a)$
Perfect square (sum)	$x^2+2ax+a^2=(x+a)^2$
Perfect square (difference)	$x^2-2ax+a^2=(x-a)^2$
General trinomial	$x^2+(a+b)x+ab=(x+a)(x+b)$
Sum of cubes	$x^3+a^3=(x+a)(x^2-ax+a^2)$
Difference of cubes	$x^3-a^3=(x-a)(x^2+ax+a^2)$

Pattern	Factors as
Difference of even powers	$x^{2n} - a^{2n} = (x^n - a^n)(x^n + a^n)$

Solving Quadratics

Method	Statement
Quadratic Formula	For $ax^2 + bx + c = 0$ ($a \neq 0$): $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Discriminant > 0	Two distinct real solutions
Discriminant $= 0$	One repeated real solution
Discriminant < 0	Two complex solutions
Square Root Property	If $x^2 = p$, then $x = \pm\sqrt{p}$

Completing the square, worked example. Solve $3x^2 + 6x - 24 = 0$.

1. Divide every term by the leading coefficient 3: $x^2 + 2x - 8 = 0$
2. Move the constant to the other side: $x^2 + 2x = 8$
3. Halve the x-coefficient, square it, add to both sides: $x^2 + 2x + 1 = 8 + 1 = 9$
4. Factor the left side: $(x + 1)^2 = 9$
5. Apply the Square Root Property: $x + 1 = \pm 3$
6. Solve for x: $x = -1 \pm 3$, so $x = 2$ or $x = -4$

Logarithm Rules

Rule	Statement
Definition	$y = \log_b(x)$ means $x = b^y$
Worked example	$\log_2(64) = 6$, because $2^6 = 64$
Identity values	$\log_b(b) = 1$, $\log_b(1) = 0$
Inverse of an exponential	$\log_b(b^x) = x$, $b^{\log_b(x)} = x$
Power Rule	$\log_b(x^r) = r \log_b(x)$

Rule	Statement
Product Rule	$\log_b(xy) = \log_b(x) + \log_b(y)$
Quotient Rule	$\log_b\left(\frac{x}{y}\right) = \log_b(x) - \log_b(y)$
Natural and common log	$\ln(x) = \log_e(x)$; $\log(x) = \log_{10}(x)$
Domain	$\log_b(x)$ is only defined for $x > 0$

Lines and Function Forms

Form	Equation
Slope of a line through two points	$m = \frac{y_2 - y_1}{x_2 - x_1}$
Slope-intercept form	$y = mx + b$, y-intercept $(0, b)$
Point-slope form	$y = y_1 + m(x - x_1)$
Parabola, vertex form	$y = a(x - h)^2 + k$, vertex (h, k)
Parabola, standard form	$y = ax^2 + bx + c$, vertex at $x = -\frac{b}{2a}$
Distance between two points	$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Conic Sections

Shape	Equation
Circle	$(x - h)^2 + (y - k)^2 = r^2$, center (h, k), radius r
Ellipse	$\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$
Hyperbola (opens left/right)	$\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1$
Hyperbola (opens up/down)	$\frac{(y - k)^2}{b^2} - \frac{(x - h)^2}{a^2} = 1$

Common Algebra Mistakes

What goes wrong	Why, with an example
$\frac{5}{0}$ treated as 0 or as 5	Division by zero is undefined — it isn't any number at all.
$-4^2 = 16$	Without parentheses, the exponent binds to 4 only: $-4^2 = -16$, while $(-4)^2 = 16$.
$(x^3)^4 = x^7$	A power of a power multiplies exponents: $(x^3)^4 = x^{12}$, not x^7.
$\frac{a}{b+c} = \frac{a}{b} + \frac{a}{c}$	Check with numbers: $\frac{1}{2+3} = \frac{1}{5}$, but $\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$ — not equal.
$\frac{a+3x}{a} = 1+3x$	Only a cancels: $\frac{a+3x}{a} = 1+\frac{3x}{a}$; the x-term keeps its denominator.
$-2(x-5) = -2x - 5$	The negative sign has to distribute to both terms: $-2(x-5) = -2x+10$.
$(x+3)^2 = x^2+9$	Expanding fully: $(x+3)^2 = x^2+6x+9$ — the middle term is real.
$\sqrt{5^2+12^2} = 5+12$	$\sqrt{25+144} = \sqrt{169} = 13$, not 17 — a square root doesn't distribute over a sum.
$3(x+2)^2 = (3x+6)^2$	Squaring comes first: $3(x+2)^2 = 3x^2+12x+12$, while $(3x+6)^2 = 9x^2 + 36x + 36$.
$a^0 = 0$	Any nonzero base to the zero power is 1, not 0.